Optimizing AUV Deployment using Finite-Time Lyapunov Exponents from Ocean Flow Data Off the Irish Coast

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Background

Approximately 97% of all global data passes through undersea cables: fiber-optic cores that facilitate emergency services, commerce, and military operations worldwide, and Ireland's position off the mainland of Europe makes it so 75% of transatlantic cables travel through Irish waters.

This presents an attractive opportunity for military or criminal activity. After an incident in November 2024 where a Russian intelligence ship was spotted operating near undersea cables south of Cornwall, it became clear that Ireland does not have the appropriate Navy to protect against this vulnerability. Therefore, it's critical to find a method to monitor Irish waters in a way that's both low-cost and easy to maintain.

Introduction

The goal of this project was to analyze available marine data to determine where to deploy AUV's such that they would gather sensor data with a wide range of coverage off the Irish coast.

To do this, we used ocean data from the Connemara 2D Oceanographic model for Galway Bay. This included Barotropic Sea Water velocity data in the latitude and longitude directions, as well as a time variable.

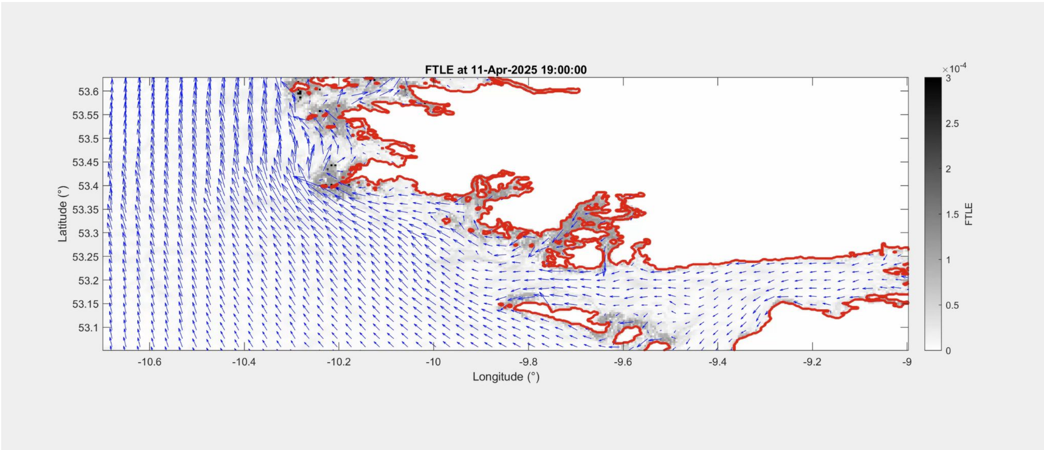

Figure 1. Reverse Time FTLE, showing Attraction

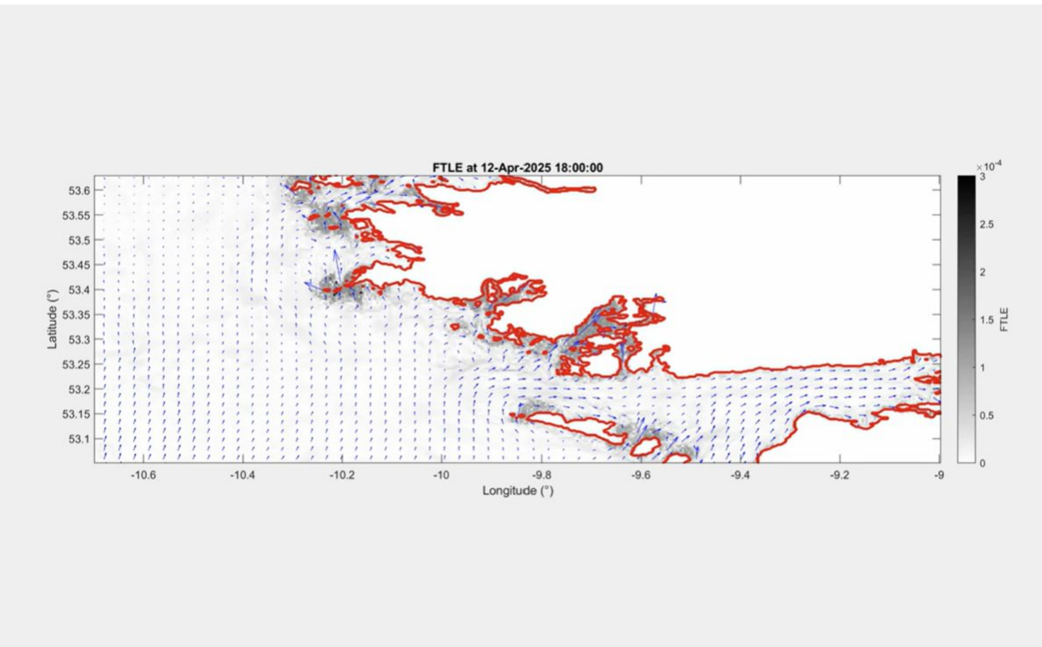

Figure 2. Eddys and Particle Accumulation in the Direction of Inner Bays

Methods

Lagrangian Coherent Structures (LCS) describe how particles move in unsteady fluids. They are characterized by stable and unstable manifolds, which create "hills" and "valleys" that attract and repel particles, respectively.

To calculate the LCS, we must first calculate the position of a passive particle moving through an unsteady fluid flow (we treat the ocean as a vector field). This produces the equation

$$\frac{d}{dt}x = u(x,t)$$

where u(x,t) represents the vector field.

We use MatLab to map this grid of particles forward through time. Using this data, we create the Flow Map Jacobian, a map that tracks how much a particle is moving in the x and y directions. Using this we can identify places with a large Lyapunov exponent, essentially the maximum distance close particles spread through a flow field in a given time. These regions of high stretching and deformation help us identify where the unstable and stable manifolds are. To find the FTLE, we use the equation

$$\Lambda_{t_0}^{t_1}(x_0) = \frac{1}{t_1 - t_0} \log \sqrt{\lambda_n(x_0)}$$

Regions with high FTLE values correspond to unstable manifolds and regions with low FTLE correspond to stable manifolds.

This is all done using code and the LCS tool from the George Haller group. First we have MatLab interpolate data for time and space for particles in each x and y position. We then iterate through timesteps using the LCS tool. We compute the Cauchy-Green strain tensor eigenvalues/vectors for each hour (because the Connemara data is broken into hours) and calculate the FTLE field using the maximum eigenvalue. Then the system shows the FTLE contours with a grid of background vectors (and a land mask, to show the coast).

Conclusion

The high value of the vectors within and near the bay mouths indicate that they may be a hazardous landscape for the unmanned underwater vehicle to navigate closely to. Even though the attraction does cycle through alternate directions (which would push the vehicle in and out of each bay over the course of the day), the strength of the current and the proximity to rock means the vehicle may get damaged or simply wash up ashore. Care would need to be taken to ensure the vehicle remained appropriately distant from the coast. Additionally, the data indicates that 19:00-22:00 could be a daily optimal time for launch, as the vector field points away from shore at that time over the course of the three days.

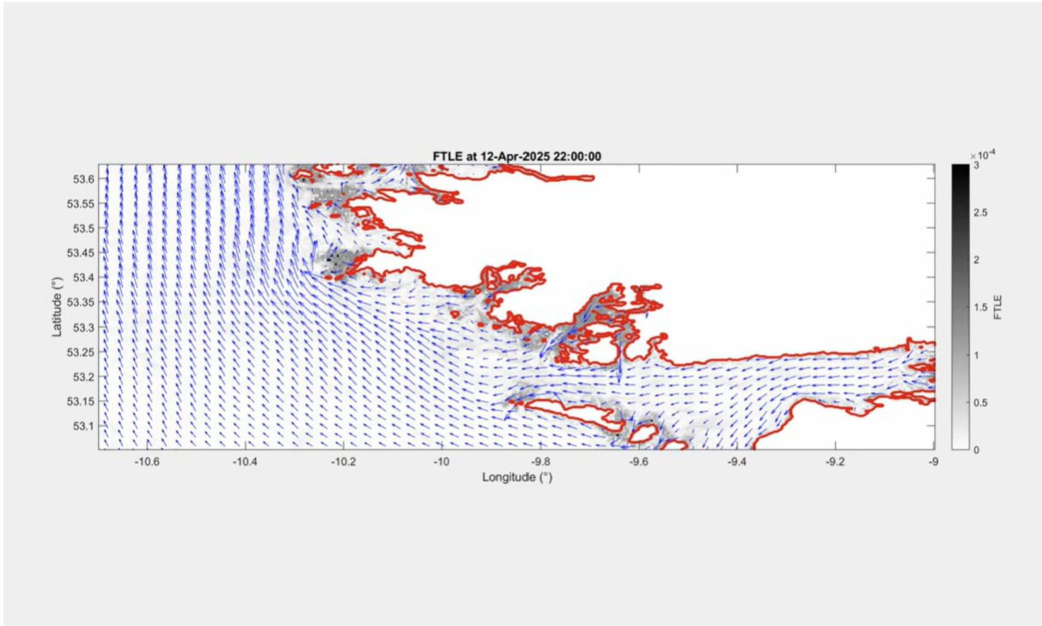

Figure 3: Fluid Flow Away from the Mouth of Bays

Results

[Figure #1] shows where particles are attracted. It shows where particles most distant got closest together in forward time, by determining which particles diverged the most in backward time.

The FTLE fields computed from the galway coast's data reveal some distinct patterns, but further analysis is required to fully interpret where the unstable and stable manifolds are at each point in time. Simple analysis shows that attracting LCSs form within the inner bay and areas with deep inner coastlines at certain times.
[See Figure #2]

This is consistent with "recirculation zones" of sorts seen at other coastlines, which are essentially circular currents caused by wind patterns combined with the shape of the coast. The vector field points steeply into these deep bays, and forms a circular pattern between the two in the center.

We can also see from this data that repelling LCS appear near the bay mouth, and between recirculation zones.
[See Figure #3]
Fluid Flow Away from the Mouth of Bays
At this time, we can see the vector field pointing away from the mouth of the bay. This is approximately 27 hours after the image from Figure 1, which shows a similar pattern of attraction out and away from the bay mouths.

Further work

Further computational analysis is required to fully understand and visualize the LCS structures within the Connemara data. Also, an additional set of code would be required to calculate the best path for an unmanned underwater vehicle to travel in order to get the most coverage off the western coast. The vehicle would need to utilize both attracting LCSs to move along the ocean's surface and repelling LCSs to avoid drifting out to sea.

Experimentation and testing would be necessary to determine the most accurate and least intensive way of generating potential routes.

The 72 hour scale for this project is also relatively brief. Silbo's team found an 8 day scale to be sufficient to provide a "detailed landscape" to guide the vehicle, and by looping that 8 days we could generate routes over any period of time.

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